An Adaptive FEM Based on Magnetic Field Conservation Applying to Ferromagnetic Problems

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We have previoulsy proposed a novel adaptive mesh refinement method based on magnetic field conservation and a non-conforming mesh refinement technique. In the proposed method, the magnetic field conservation on element interfaces is evaluated as an error indicator, and the non-conforming finite element technique is employed as a mesh refinement scheme. Combining these techniques, the proposed method can improve the simulation accuracy with less number of finite elements. However, we have applied to a very simple model consisting of a single permanent magnet.

The proposed method has two advantages: (1) it is easy to evaluate the magnetic field conservation on surfaces between different materials, and (2) it is easy to subdivide elements with large error on surfaces between different materials into smaller elements. Therefore, the proposed method is easily applicable to complicated model containing plural kinds of materials. In this paper, we have tried to apply the new adaptive meshing method to a more complicated model, where ferromagnetic material with non-linear property is included.

Index Terms—**Adaptive finite element method, error estimation, mesh refinement, non-conforming finite element method.**

I. Introduction

We have previously proposed a novel adaptive finite element method (FEM) utilizing a magnetic field conservation evaluation as an error indicator and a non-conforming mesh refinement technique as a mesh refinement [1]. Our concept is to improve the simulation accuracy with less number of elements. Though the performances of PCs are enhanced, the generation of an unnecessary large number of elements is undesirable in an adaptive FEM.

Although some error indicators were proposed [2]-[5], an error indicator based on the conservation of magnetic field $\boldsymbol{H}$ at the interface between two elements [5] are very promising from the mathematical viewpoint.

Meanwhile, a few kinds of non-conforming techniques were also proposed such as the discontinuous Galerkin method [6], the mortar FEM [7], and the mesh interpolating method [8]. The interpolating method is well-suited for an adaptive FEM.

The proposed adaptive FEM employing these two techniques resulted with the generation of a suitably coarse mesh with less number of elements. The proposed method has two advantages: (1) it is possible to indicate an error on surfaces between different material elements, and (2) it is easy to subdivide elements on the surfaces of objects. That is, it is easily applicable to a simulation model including iron cores or plural kinds of materials. However, we have never shown any result of such models, we present a result applying the proposed adaptive FEM to a model consisting of a single permanent magnet and an iron core.

II. Adaptive Edge-based Finite Element Method

Fig. 1 shows the flow of adaptive FEM. In this paper, a edge-based tetrahedral finite element is employed.

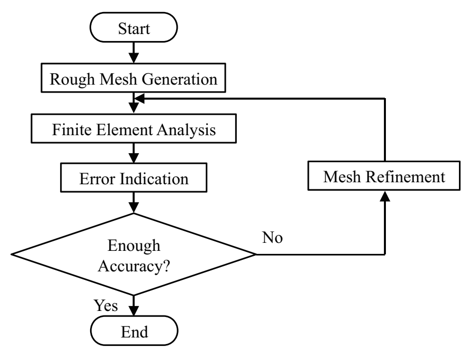

Fig. 1. Flow of traditional adaptive finite element method.

A. Error Indicator Base on Magnetic Field Conservation

For an error indicator, a weighted tangential component of magnetic field, d, is obtained:

$$d_{i,j} = \int_S (\boldsymbol{H}_i \times \boldsymbol{w}_j) \cdot \boldsymbol{n}_i \mathrm{d}S \qquad (i = 1, 2 \text{ and } j = a, b, c) \quad (1)$$

where i, j, S, $\boldsymbol{H}$, $\boldsymbol{w}$, and $\boldsymbol{n}$ are the indexes of adjacent elements and edges (see Fig. 2), the surface of element, the magnetic field, the vector interpolation function, and the unit vector normal to the element surface S, respectively. Due to the magnetic field conservation, the following equations with respect to all three edges per element surface have to hold:

$$D_j = d_{1,j} + d_{2,j} \qquad (j = a, b, c) \quad (2)$$

As a result of the ordinary edge-based FEM, the values for D_j $(j = a, b, c)$ are not zero. Therefore, as an error indicator, we employ the following equation:

$$E_{i,j} = \max\left(|D_a|, |D_b|, |D_c|\right) \quad (3)$$

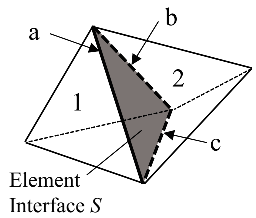

Fig. 2. Adjacent tetrahedral elements 1 and 2 with common edges a, b, c.

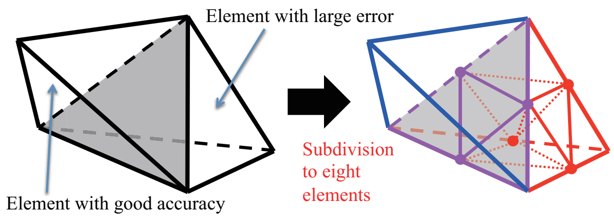

Fig. 3. Subdivision from an element with large error to eight smaller elements.

Because the tangential component of magnetic field $\boldsymbol{H}$ is continuous on surfaces between different materials, this error indicator is useful, robust, and effective.

B. Mesh Refinement Utilizing Non-conforming FEM

Mesh refinement is a burdensome task. As a mesh-making method, the Delaunay triangulation method is conventionally used. However, many ill-quality elements, such as flat or inside-out elements, are generated with adaptive steps.

As shown in Fig. 3, in the proposed mesh refinement scheme, only elements indicated with a large error are subdivided into eight smaller elements. Using the non-conforming finite element technique, a special mesh-making technique is not needed, and the subdivision scheme is easily applied to elements on the boundary of analysis objects.

III. Application

The proposed adaptive method is robust to models consisting of plural kinds of materials, such as iron. To show the effectiveness, the proposed adaptive FEM is applied to a model consisting of a permanent magnet and an iron, as show in Fig. 4. The initial mesh of 18,838 elements was created by commercial software. The magnetic energy error of the initial mesh is 7.48%.

Fig. 5 presents the magnetic energy error as a function of number of elements. In the first few steps, the error is drastically reduced. At the 5th adaptive step, the error decreased to 0.06% with 708,989 elements.

Fig. 6 shows the flux line maps of the final FEM result. The proposed adaptive method functions as a smoother of flux lines by improving the discontinuity of the tangential component of magnetic field. Every flux line in Fig. 6 looks like enough smooth.

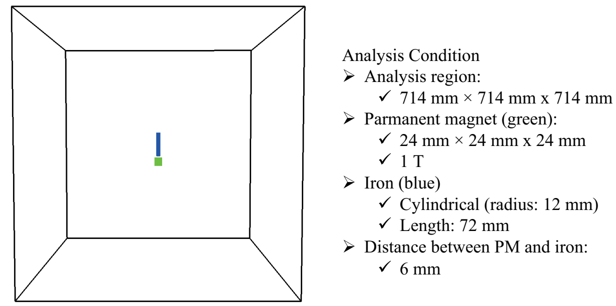

Fig. 4. Model consisting of one permanent magnet and an iron core.

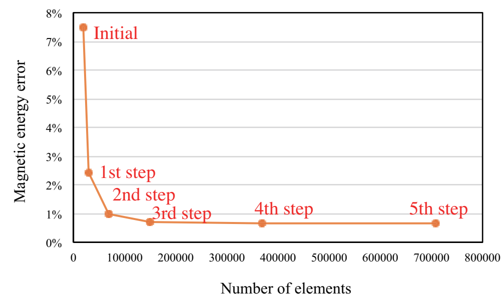

Fig. 5. Model consisting of one permanent magnet and an iron core.

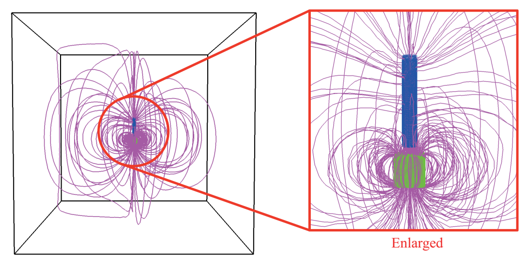

Fig. 6. Magnetic flux lines computed from the final adaptive meshing result.

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